For, bisecting FE in G and drawing AG, $CD=FE=2AB$ according to the nature of the conchoid. Since $FG=GE=AG=AB$, we easily find $\angle CBE=\frac{1}{3}\angle ABC$. By the lower conchoid $C'E'$, the obtuse adjacent angle is trisected. For, bisecting $E'F'$ ($=C'D=2AB$) in G' and drawing AG', we have $E'G'=G'A=G'F'=AB$, from which at once follows $\angle E'BC'=\frac{1}{3}\angle ABC'$.

Nicomedes devised the conchoid for the trisection of an angle and the duplication of a cube.

3. Let BEC represent an Archimedean Spiral. Divide the radius BC of the circular arc AC, into three equal parts so that $BD=\frac{1}{3}BC$, then, describing from B, with radius BD, an arc which intersects the spiral at E, the angle $ABE=\frac{1}{3}$ angle ABC. For, according to the definition of the spiral, $AB : BE(=BD)=\angle ABC : \angle ABE$.

4. *Montucla* ascribes the following two solutions to the Platonic school.

1. Let ACB be the angle to be trisected. From C, with any radius, describe a semi-circle, and through B draw BE, interseting the circle in D, so as to make $DE=$ the radius of the circle, then angle at $E=\frac{1}{3}ACB$.

2. Let ABC be the angle to be trisected. Complete the rectangle above BC. Produce the upper side, and through B draw BE meeting the upper side produced in E and intersecting the perpendicular CA in D, so as to make $DE=2AB$, then angle $DBC=\frac{1}{3}ABC$, as can be easily proved by drawing AG to the middle point of DE.

5. The jesuit *Thomas Ceva* devised an instrument for the trisection of an angle. It consists of four rulers, AE, AF, DB, DC, which form a rhombus, $ABDC$, and are movable around A, B, C, D. (The points B, G and C, H being, respectively, on AE and AF.) If the angle GDH is to be trisected, we take $GD=DH=BD$, fasten the instrument at D, and move the rulers so as to make AE and AF pass through G and H, then angle $EAF=\frac{1}{3}GDH$.

6. By approximation we can trisect the angle $BCA=a$, in the following manner:

Make $AD=\frac{1}{4}a$, $DE=\frac{1}{4}AD=\frac{1}{4^2}a$, $EF=\frac{1}{4}DE=\frac{1}{4^3}a$, &c.; then we obtain for the sum of all these arcs, by summing the infinite geometric series $(\frac{1}{4}+\frac{1}{4^2}+\frac{1}{4^3}+\dots)a=\frac{1}{3}a$.

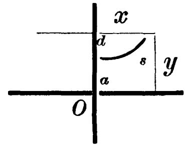
NOTE ON THE CATENARY, BY PROF. W. W. JOHNSON.—The following formulæ arise in the consideration of the measurement of a base line by means of a steel tape which is allowed to assume the form of a catenary.

The equation of the curve being

$$y=\frac{a}{2}\left(e^{\frac{x}{a}}+e^{-\frac{x}{a}}\right), \qquad (1)$$

we have
$$s=\frac{a}{2}\left(e^{\frac{x}{a}}-e^{-\frac{x}{a}}\right), \qquad (2)$$

whence
$$s^2=y^2-a^2=(y-a)(y+a). \qquad (3)$$



Now suppose the ends of the tape to be on a level, and that d, the deflection of the curve, is measured; the length of the tape is $2s$, and s and d are the known quantities. But $y-a=d$, hence from (3), $y+a=s^2\div d$; whence

$$y=\frac{s^2+d^2}{2d}, \qquad \text{and } a=\frac{s^2-d^2}{2d}. \qquad (4)$$

From (1) and (2), $$e^{\frac{x}{a}}=\frac{y+s}{a},$$

and substituting from (4),

$$e^{\frac{x}{a}}=\frac{s^2+d^2+2ds}{s^2-d^2}=\frac{s+d}{s-d};$$

whence
$$x=a\log\frac{s+d}{s-d}=\frac{s^2-d^2}{2d}\log\frac{s+d}{s-d}, \qquad (5)$$

or, putting L for $2x$, the length to be measured, and S for $2s$, the whole length of the tape,

$$L=\frac{S^2-4d^2}{4d}\log\frac{S+2d}{S-2d}, \qquad (6)$$

the logarithm being Naperian.

If we develop the logarithm by the formula

$$\log\frac{1+x}{1-x}=2x\left[1+\frac{x^2}{3}+\frac{x^4}{5}+\ldots\right],$$

we have

$$L=S\left[1-\frac{(2d)^2}{S^2}\right]\left[1+\frac{(2d)^2}{3S^2}+\frac{(2d)^4}{5S^4}+\ldots.\right]$$

$$=S\left[1-\frac{2(2d)^2}{3S^2}-\frac{2(2d)^4}{3.5S^4}-\frac{2(2d)^6}{5.7S^6}-\frac{2(2d)^8}{7.9S^8}-..\right]. \qquad (7)$$

The second term is of course the same that occurs in the case of a circular arc.

SOLUTIONS OF PROBLEMS IN NUMBER THREE.

SOLUTIONS of problems in No. 3 have been received, as follows:

From Prof W. P. Casey, 259, 264; Alexander S. Christie, 261; Geo. Eastwood, 265; W. V. McKnight, 259; W. L. Marcy, 263; E. B. Seitz, 264, 265; Prof. J. Scheffer, 259 and answer to query at page 55. Also, Prof. J. H. Kershner and R. J. Adcock each sent ans. to query at p. 55.